Time: **3hrs;** Total Marks: **100**

General Instructions:

1. All questions are compulsory
2. Please check that this question paper contains 29 questions
3. Question **1 – 4** in Section A carrying 1 mark each
4. Questions **5 – 12** in Section B carrying 2 marks each
5. Questions **13 – 23** in Section C carrying 4 marks each
6. Questions **24 – 29** in Section D carrying 6 marks each

SECTION – A

1. If for any 2 × 2 square matrix A, A (adj A) = $\begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix}$, then write the value of I A I.

2. Determine the value of 'k' for which the following function is continuous at x = 3:

$$f(x) = \begin{cases} \dfrac{(x+3)^2 - 36}{x-3}, x^{1}3 \\ k \ , x = 3 \end{cases}$$

3. Find: $\int \frac{\sin^2 x - \cos^2 x}{\sin x \cos x} dx$

4. Find the distance between the planes 2x – y + 2z = 5 and 5x – 2.5y + 5z = 20.

SECTION - B

5. If A is a skew-symmetric matrix of order 3, then prove that det A = 0.
6. Find the value of c in Rolle's theorem for the function f(x) = x^3 – 3x in [-$\sqrt{3}, 0$].
7. The volume of a cube is increasing at the rate of 9 cm 3/s. How fast its surface is are increasing when the length of an edge is 10 cm?
8. Show that the function f(x) = x^3 – $3x^2$ + 6x – 100 is increasing on R.
9. The x-coordinate of a point on the line joining the points P (2,2,1) and Q (5,1,-2) is 4. Find its z-coordinate.
10. A die, whose faces are marked 1, 2. 3 in red and 4, 5, 6 in green, are tossed. Let A be the event" number obtained is even" and B be the event "Number obtained is red." Find if A and B are independent event.
11. Two tailors, A and B earn 300 and 400 per day respectively. A can stitch 6 shirts and 4 pairs of trousers while B can stitch 10 shirts and 4 pairs of trousers per day. To find how many days should 10 shirts and if it is desired to produce at least 60 shirts and 32 pairs of trousers at a minimum labour cost, formulate this as an LPP.
12. Find $\int \frac{dx}{5 - 8x - x^2}$

SECTION – C

13. If $\tan^{-1}\frac{x-3}{x-4} + \tan^{-1}\frac{x+3}{x+4} = \frac{\pi}{4}$, then find the value of x

14. Using properties of determinants, prove that

$$\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$$

OR

Find matrix A such that

$$\begin{pmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{pmatrix} A = \begin{pmatrix} -1 & -8 \\ 1 & -2 \\ - & 22 \end{pmatrix}$$

15. If $x^y + y^x = a^b$, then find $\frac{dy}{dx}$

OR

If $e^y (x+1) = 1$, then show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$

16. Find $\int \frac{\cos\theta}{(4+\sin^2\theta)(5-4\cos^2\theta)} d\theta$

17. Find $\int_0^n \frac{x\tan x}{\sec x+\tan x} dx$

OR

Evaluate $\int_1^4 \{x-1+x-2+x-4\} dx$

18. Solve the differential equation $(\tan^{-1} x - y)\,dx = (1 + x^2)\,dy$

19. Show that the points A, B, C with position vectors $2\hat{i} - \hat{j} + k, \hat{i} - 3\hat{j} - 5k$ and $3\hat{i} - 4j - 4k$ respectively are the vertices of a right-angled triangle, Hence find the area of the triangle

20. Find the value λ, if four points with position vectors $3\hat{i} + 6\hat{j} + 9k, \hat{i} + 2\hat{j} + 3k, 2\hat{i} + 3\hat{j} + k$ and $4\hat{i} + 6\hat{j} + \lambda k$ are coplanar.

21. There are 4 cards numbered 1, 3, 5 and 7 one number on one card. Two cards are drawn at random without replacement. Let X denotes the sum of the numbers on the two cards. Find the mean and variances of X.

22. Of the students in a school, it is known that 30% have 100% attendance and 70% students are irregular. Previous year results report that 70% of all students who have 100% attendance attain a grade and 10% irregular students attain A grade in their annual examination. At the end of the year one student is chosen at random from the school and he was found to have an A grade. What is the student has 100% attendance? Is regularly required only in school? Justify your answer.

23. Maximize z = x + 2y Subject to the constraints

$x + 2y \geq 100$

$2x - y \leq 0$

$2x - y \leq 200$

$x, y \geq 0$ Solve the above LPP graphically

SECTION – D

24. Determine the product $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the system of equations

X – y + z = 4, x – 2y – 2x = 9, 2x + y + 3z = 1

25. Consider f: R $-\left\{-\frac{4}{3}\right\} \rightarrow R - \left\{\frac{4}{3}\right\}$given by f (x) = $\frac{4x+3}{3x+4}$ show that f is bijective.

Find the inverse of f and hence find $f^{-1}(0)$ and x such that $f^{-1}(x) = 2$

OR

Let A = Q × Q and let * be a binary operation on A defined by (a, b) × (c, d) = (ac, b + ad) for (a, b), (c, d) ∈ A. Determine whether * is Commutative and associative. Then, with respect to * on A

(i) Find the identify element in A.

(ii) Find the invertible elements of A.

26. Show that the surface area of a closed cuboid with square base and given volume is minimum when it is a cube.

27. Using the method of integration, find the area of the triangle ABC, coordinates of whose vertices are A(4,1), B(6,6) and C(8,4).

OR

Find the area enclosed between the parabola $4y = 3x^2$ and the straight line 3x – 2y + 12 = 0.

28. Find the particular solution of the differential equation (x-y) $\frac{dy}{dx}$ = (x+2y). given that y = 0 when x =1

29. Find the coordinates of the point where the line through the point (3,-4,-5) and (2,-3,1) crosses the plane determined by the points (1,2,3), (4,2,-3) and (0,4,3)

OR

A variable plane which remains at a constant distance 3p from the origin cuts the coordinate axes at A, B, C. Show that the locus of the centroid of triangle ABC is $\frac{1}{x^2} + \frac{1}{y2} + \frac{1}{z^2} = \frac{1}{P^2}$