CBSE XII Mathematics Board Paper 2015 Set I

Time: **3 hrs;** Total Marks: **100**

General Instructions:
1. All questions are compulsory.
2. Please check that this question paper contains 26 questions.
3. Question **1 – 6** in Section A are very short – answer type questions carrying 1 mark each.
4. Questions **7 – 19** in Section B are long – answer I type question carrying 4 marks each.
5. Questions **20 – 26** in Section B are long – answer II type question carrying 6 marks each.

Section A

1. If $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + 5\hat{j} - 2\hat{k}$,then find $|\vec{a} \times \vec{b}|$.

2. Find the angle between the vectors $\hat{i} - \hat{j}$ and $\hat{j} - \hat{k}$.

3. Find the distance of a point (2 , 5 , -3) from the plane $\vec{r}.(6\hat{i} - 3\hat{j} + 2\hat{k})$ = 4.

4. Write the element a_{12} of the matrix A = $[a_{ij}]_{2\times 2}$, whose elements a_{ij} are given by $a_{ij} = e^{2ix}$ sinjx.

5. Find the differential equation of the family of lines passing through the origin.

6. Find the integrating factor for the following differential equation: $x \log x \frac{dy}{dx} + y = 2 \log x$

Section B

7. If A = $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then show that A^2 - 4A -5I = 0, and hence find A^{-1}.

OR

If A = $\begin{vmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{vmatrix}$, then find A^{-1} using elementary row operations.

8. Using the properties of determinants solve the following for x: $\begin{vmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix}$ = 0.

9. Evaluate: $\int_0^{\Pi/2} \frac{\sin^2 x}{\sin x + \cos x} dx$.

OR

Evaluate $\int_{-1}^{2} \left(e^{3x} + 7x - 5\right) dx$ as a limits of sum.

10. Evaluate: $\int \frac{x^2}{x^4 + x^2 - 2} dx$

11. In a set of 10 coins, 2 coins are with heads on both the sides. A coin is selected at random from this set and tossed 5 times .If all the 5 times, the result was heads, find the probability that the selected coin had heads on both the sides.

OR

How many times must a fair coin be tossed so that the probability of getting at least one head is more than 80%?

12. Find x such that the four points A(4,1,2), B(5,x,6) ,C(5,1,-1) and D(7,4,0) are coplanar.

13. A line passing through the point A with position vector $\vec{a} = 4\hat{i} + 2\hat{j} + 2\hat{k}$ is parallel to the vector $\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ .Find the length of the perpendicular drawn on this line from a point P with vector $\vec{r}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$.

14. Solve the following for x:

$$\sin^{-1}(1-x) - 2\sin^{-1} x = \Pi / 2$$

OR

Show that:

$$2\sin^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{17}{31}\right) = \Pi / 4$$

15. If y = e^{ax}.cosbx, then prove that

$$\frac{d^2 y}{dx^2} - 2a\frac{dy}{dx} + \left(a^2 + b^2\right)y = 0$$

16. If $x^x + x^y + y^x = a^b$, then find $\frac{dy}{dx}$.

17. If $x = a\sin 2t(1+\cos 2t)$ and $y = b\cos 2t(1-\cos 2t)$ then find $\frac{dy}{dx}$ at t=∏/4.

18. Evaluate: $\int \frac{(x+3)e^x}{(x+5)^3}dx$

19. Three schools X,Y and Z organized a fete(mela)for collecting funds for flood victims in which they sold hand -held fans, mats and toys made from recycled material ,the sale price of each being Rs.25 , Rs.100 and Rs.50 respectively. The following table shows the number of articles of each type sold:

School/Article	School X	School Y	School z
Hand -held fans	30	40	35
Mats	12	15	20
Toys	70	55	75

Using matrices, find the funds collected by each school by selling the above articles and the total funds collected. Also write any one value generated by the above situation.

Section C

20. Let A = Q×Q , where Q is the set of all rational numbers , and * be a binary operations on A defined by

(a,b) * (c,d) =(ac,b +ad) for (a,b) ,(c,d)∈A.Then Find

(i) the identity element of * in A.

(ii) Invertible elements of A, and hence write the inverse of elements (5,3) and (1/2 ,4).

OR

Let f: W→W be defined as

F(n) = { n−1,if n is odd

{n+1,if n is even

Show that f is invertible and find the inverse of f.Here, W is the set of all whole numbers.

21. Sketch the region bounded by the curves $y = \sqrt{5-x^2}$ and $y = |x-1|$ and find its area using integration.

22. Find the particular solution of the differential equation $x^2dy = (2xy + y^2)dx$ given that y = 1 when x = 1.

OR,

Find the particular solution of the differential equation

$$\left(1+x^2\right)\frac{dy}{dx}=\left(e^{m\tan^{-1}x}-y\right)$$,given that y = 1when x =0.

23. Find the absolute maximum and absolute minimum values of the function f given by f(x) =$\sin^2$x-cosx, $x\in\left(0,\Pi\right)$

24. Show that lines :

$$\vec{r}=\hat{i}+\hat{j}+\hat{k}+\lambda\left(\hat{i}-\hat{j}+\hat{k}\right)$$

$$\vec{r}=4\hat{j}+2\hat{k}+\mu\left(2\hat{i}-\hat{j}+3\hat{k}\right)$$ are coplanar.

Also, find the equation of the plane containing these two lines.

25. Minimum and maximum constraints z = 5x + 2y subject to the following constraints:

$x - 2y \le 2$

$3x + 2y \le 12$

$-3x + 2y \le 3$

$x \ge 0,\ y \ge 0$

26. Two of the numbers are selected at random (without replacement from the first six positive integers .Let X denote the larger of the two numbers obtained .Find the probability distribution of X .Find the mean and variance of the distribution.